Exponential Equations:

$y = x^2$ is a power function. (ie. y = x to the second power)

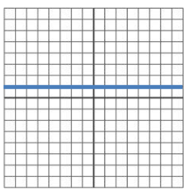
$y = 2^x$ is an Exponential function.

We will use equations of the form $y = a \cdot b^x$, where a and b are constants.

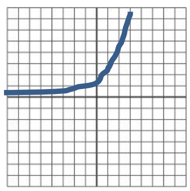
b is called the base   x is called the exponent

The graph of exponential functions look like:

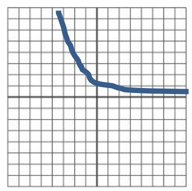
$y = 1^x$



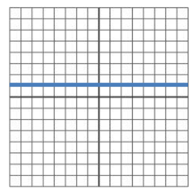
$y = 2^x$



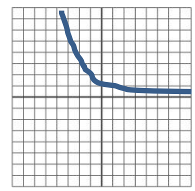
$y = \left(\frac{1}{2}\right)^x$



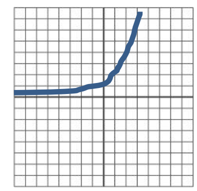
$y = 1^{-x}$



$y = 2^{-x}$



$y = \left(\frac{1}{2}\right)^{-x}$



For $y = b^x$,

If b > 1, it is exponential GROWTH.

If 0 > b > 1, it is exponential DECAY.

For $y = b^{-x}$,

If b > 1, it is exponential DECAY.

If 0 > b > 1, it is exponential GROWTH.

The basic model we will use is $y = a \cdot b^x$,   where y is the dependent variable,

a = the initial value (x=0),   b = growth factor,   x = independent variable (usually time).

Ex 1: Write an exponential equation that goes through the points (0, 4) and (5, 30).

The model is $y = ab^x$. Plugging in the point (0, 4) → $4 = a \cdot b^0 = a \cdot 1 = a$
So the model is now $y = 4b^x$. Plugging in the other point (5, 30) → $30 = 4 \cdot b^5$
Divide both sides by 4: $7.5 = b^5$
Take the fifth root: $\sqrt[5]{7.5} = \sqrt[5]{b^5}$ → $1.496 = b$ So the equation is $y = 4 \cdot 1.496^x$

Ex 2: You have 20 bacteria in a Petri dish. The number of bacteria doubles every day.
A. Write an equation that models this situation. $b = 20 \cdot 2^t$
B. How many bacteria will there be in 1 week?
$b = 20 \cdot 2^7 = 2560$ (note 1 week = 7 days)

Ex 3: In 2000, the population of Mud County was 50,000. In 2003 the population was 75,000.
Write an equation that models this situation.
$P = 50000 \cdot b^t$ → In 2003, t = 3 (years later) and P = 75000 → so $75000 = 50000b^3$
Dividing: $1.5 = b^3$ → Cube root: $b = 1.145$ → so $P = 50000(1.145)^t$